**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**ANS) B**

**LET TIME TAKEN FOR SERVICE TRANSMISSION= T**

**T IS NORMALLY DISTRIBUTED WITH *μ* = 45 MINUTES AND STANDARD DEVIATION *σ* = 8 MINUTES.**

**TIME DELAY= 10 MINUTES**

**TIME AVAILABLE TO FINISH THE WORK= 60-10=50 MINUTES.**

**THEREFORE FROM THE EQUATION Z=(T-µ)/ *σ***

**P(T≤50)=P(Z≤(50-45)/8)=P(Z≤0.625)= 0.7324(USING Z TABLE)**

**THEREFORE P(T>50)=1-P(≤50)= 1-0.7324= 0.2676**

**(OR)**

**USING R-FUNCTION : [1-PNORM(50,45,8)]**

**OR PYTHON USING BELOW:**

|  |
| --- |
| **> 1-STATS.NORM.CDF(50, LOC =45, SCALE = 8 )**  **0.2659855** |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS)A)FALSE 68% OF THE DATA FALLS WITHIN ONE STANDARD DEVIATION OF THE MEAN (µ+*σ)*. HERE µ=38, *σ* =6, THEN, µ+*σ= 38+*6=44**

**ANS)B)TRUEZ=(X-µ)/ *σ***

**P(X≤30)=P(Z≤(30-38)/6)=P(Z≤-1.33)= 0.0918(USING Z TABLE)**

**EXPECTED COUNT=0.0918\*400= 36.72**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**X1 AND X2 ARE TWO SEPRATE RANDOM VARIABLE**

**2 X1 MEANS, X1=X2, X=X1+X2 , X=2X1 BOTH ARE EQUAL**

**ANS) 2 IS SIMPLY A LARGER SCALE VERSION OF THE RANDOM VARIABLE *X1.* IF IS NORMALLY DISTRIBUTED THEN 2X1 IS ALSO NORMALLY DISTRIBUTED.**

***X*1 AND *X*2 ARE NORMAL DISTRIBUTED, THE ASSOCIATED SUMS AND RANDOM SAMPLES ARE EXACTLY (AND NOT JUST APPROXIMATELY) NORMAL, WITH THE APPROPRIATE PARAMETERS.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**ANS)**

**HERE WE NEED RANGE OF 99% DATA WHICH LIES BETWEEN 3RD STANDARD DEVIATION OF THE MEAN.**

**HERE µ=100, *σ* =20**

**FROM EMPIRICAL RULE, µ±3*σ= 100±3\*20=>(100-60, 100+60)=>(40,160).***

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer:**

let, X is the sum of two random variables having normal distribution.

E[X] = E [45\*(profit 1+profit 2)]= 45\*(5+7)=540 million rupees

SD[X] = SD [profit 1 +profit 2]=> 45\*()

= 45\*= 225 million rupees.

Therefore, **X~ N (540,)**

1. From the empirical rule, approximately 95% of the data falls within two standard deviation of the mean.

μ ± 2σ = 540±2\*225=> (540-450, 540+450) => **(90,990)**

1. to find 5th percentile from the left side we can use the formula,

μ - 1.5σ => 540-(1.5\*225) =>**202.5 million rupees.**

1. From Q we have to find original profit distributions.

Z score for a profit of zero: Z=(X-µ)/ *σ =>* (0-5)/3 => -1.66=0.0485

Or

Devision1: stats.norm.cdf(0,5,3)= **0.0477903522728147**

Devision2: stats.norm.cdf(0,7,4)= **0.040059156863817086**

Probability of making more loss by **division 2**